The Image Ladder

A Graphical Technique to analyze Sub-Nyquist Sampling Digital Receivers

Until recently, digital receiver designs have assured non-aliased sampling of the analog input signal by sampling at or above the Nyquist rate. For example, if the front-end filter passed significant energy at frequencies up to 170 MHz, the A-to-D converter would sample at a rate of at least 340 MSps. While this conversion speed is possible with available converters, using a high sample rate increases the cost of the converter and the associated digital logic.

In order to reduce the sample rate required for a bandpass digital receiver, under-sampling or sub-Nyquist sampling can be used. Sub-Nyquist sampling in a bandpass receiver samples at a speed related to twice the signal bandwidth (rather than twice the highest frequency) and aliases the input signal to a frequency band consistent with the true Nyquist frequency. For a receiver with 160 MHz IF with a 10 MHz bandwidth such as the one described in Harris Application note AN9658[1], the sample rate must accommodate only the 10 MHz IF bandwidth rather than the 160 MHz intermediate frequency. In structuring a sub-Nyquist sampling receiver, it is necessary to set the sample rate and input filter bandwidth to provide an aliased signal at a usable frequency. This setting prevents the desired spectrum from being folded upon itself and prevents undesirable signals from interfering with the desired signal.

The Image Ladder analysis technique was developed to depict on a graph the relationship between input frequency, sample rate and sampled or aliased frequency. Figure 1 shows an Image Ladder. On the left side of the figure two vertical bars represent DC and the Nyquist frequency associated with the specified sample rate. The zigzag line between the end bars shows all possible analog input frequencies. Increasing input frequencies ascend the zigzag line. These input frequencies are marked on the vertical end bars. Projecting a point from the zigzag line to the horizontal line at the base of the ladder shows the corresponding aliased frequency (AF) represented by the sampled signal. Each time the input frequency exceeds a multiple of the Nyquist frequency it is reflected back so that it is always aliased within the Nyquist bounds of DC and one-half the sample rate Fs/2. As input frequency progresses up the ladder toward the right, the frequency of the aliased signal increases. An increase in input frequency moving up and to the left decreases the aliased signal frequency. All frequencies represented by the intersection of a vertical line and the zigzag will be aliased to the same sampled signal frequency. The corresponding aliased frequency is marked by the intersection of the vertical line and the horizontal aliased frequency scale.

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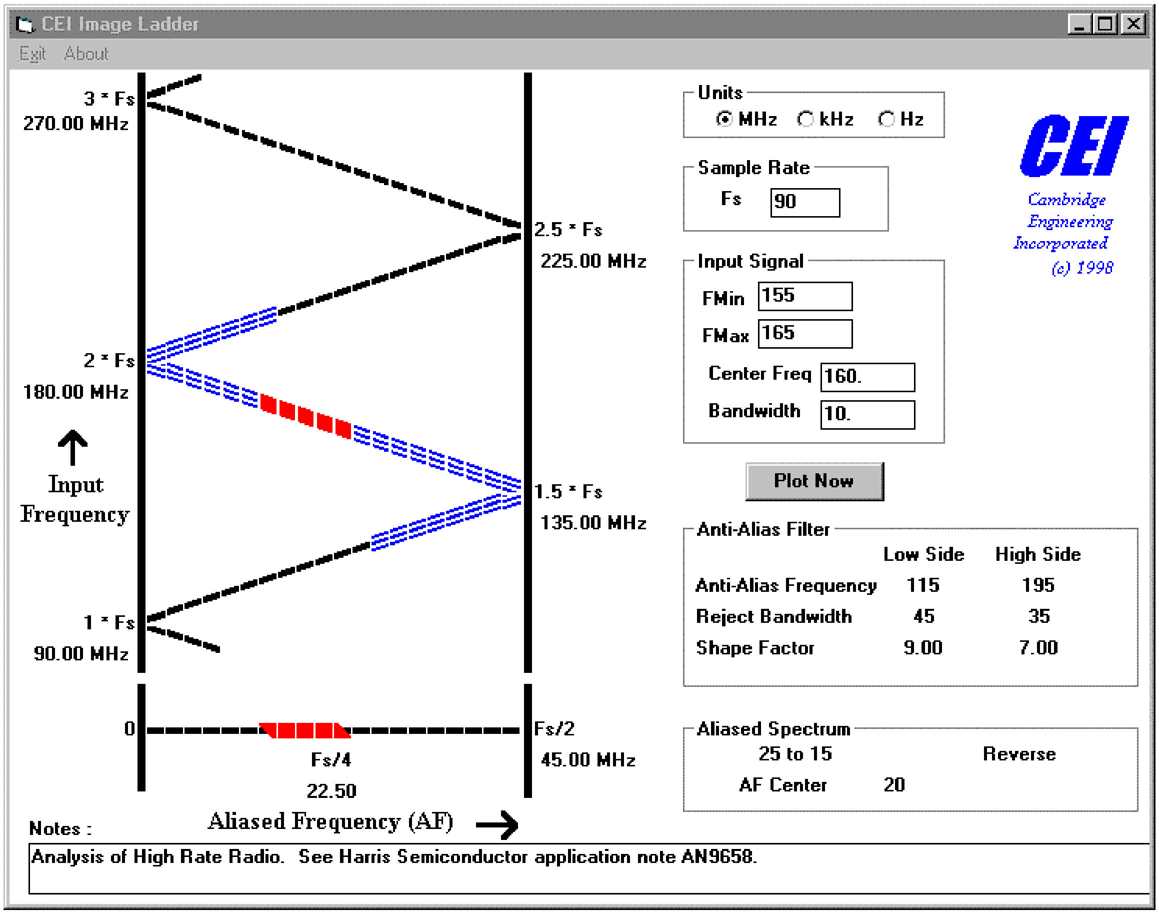
[1] For example, see Harris Semiconductor Application Note AN9658.
http://www.semi.harris.com/data/an/an9/an9658/an9658.pdf



Units
● MHz ○ kHz ○ Hz

Sample Rate
Fs 90

Input Signal
FMin 155
FMax 165
Center Freq 160.
Bandwidth 10.

Anti-Alias Filter

	Low Side	High Side
Anti-Alias Frequency	115	195
Reject Bandwidth	45	35
Shape Factor	9.00	7.00

Aliased Spectrum
25 to 15 Reverse
AF Center 20

Notes :
Analysis of High Rate Radio. See Harris Semiconductor application note AN9658.

Figure 1 Spectrum for Direct Digital Receiver

Marking the range of desired input frequencies on the zigzag line provides several pieces of information. First, projecting the input frequencies onto the aliased frequency axis allows one to determine the resulting range of aliased frequencies. Secondly, one can find input frequencies that cause undesired aliasing by projecting the desired aliased frequency range back up to the zigzag, which also exposes those frequency bands that must be removed by an anti-alias filter before sampling. Finally, one can find the shape factor for the required anti-alias filter from the bandwidth of the desired signal spectrum and the distance to the closest frequency band to be rejected.

The zigzag line depicts three frequency regions. Starting at the bottom, a broad dashed line shows frequencies that should be rejected by the anti-alias filter. As the input frequency increases, a triple dashed line shows frequencies in a transition region where the filter may start to reject signals without interfering with the specified operating band of the receiver. A solid block dashed line marks the third region and represents the desired operating band of the receiver. The input filter should pass frequencies in this region with minimum attenuation. As the input frequency continues to increase, a second transition zone is shown. This zone is followed by a second region of maximum filter attenuation.

The construction of the Image Ladder is simplified by a Microsoft Visual Basic® program that draws an image ladder based on data entered on the right side of the screen. Inputs are the frequency units (MHz, kHz or Hz), the sample rate and limits of the input frequency band. Input signal limits can either be entered as the minimum and maximum frequencies or as center frequency and bandwidth. Numeric outputs show the nearest frequencies that will alias onto the desired signal. Bandwidth and shape factor of the required analog anti-alias filter are also shown in this box. Because the requirements for high and low side rejection are not symmetrical, shape factors are given for both high and low side. The Aliased Spectrum box shows frequency limits and center frequency of the desired aliased spectrum.

Figure 1 shows the results of using the Image Ladder to evaluate the radio presented in AN9658. The sample rate of 90 MHz and a 10 MHz passband specified in this Application Note give the results shown. The Image Ladder on the left hand side of the figure shows how the aliasing property of a sampling A-to-D converter simultaneously samples and reduces the frequency of the 160 MHz IF signal to a convenient alias frequency (AF) band centered on 20 MHz. Examination of the Image Ladder also shows other input frequency bands that will appear on the desired AF band if they are not removed by the IF filter. The frequency bands that constrain the IF filter design are centered on 110 MHz and 200 MHz. The box labeled “Anti-Alias Filter” displays the upper and lower edges of these frequency bands. The anti-alias filter analysis indicates that frequencies below 115 MHz and above 195 MHz must be attenuated enough to prevent interference with the desired signals. The diagram also displays the Fs/4 frequency and shows its relationship to the aliased input signal.

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